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Citation of PhD Theses

We plan to cite theses accepted for PhD in AMO Physics in India. We believe that this will keep the community updated about the work undertaken by young researchers and offer an opening for further opportunities.

Supervisors may send us:

i) *Title of the thesis*
ii) *Name of the Candidate*
iii) *University/Institute*
iv) *Date of submission*
v) *Name of supervisor*
vi) *Synopsis of the thesis*

We will accept backdated theses with a cutoff date 01-01-2006.

Editor

FROM THE EDITOR'S DESK

Creation of atomic coherence, which leads to interesting phenomena in quantum optics, till date relies on monochromatic lasers. This may change in future, the news article reports theoretical studies demonstrating the possibility of producing multicolour atomic coherence.

There are two abstracts on the studies of oxygen molecule. These perhaps compensate for the hazardous cocktail of molecules covered in the previous issue of the newsletter: carbondioxide and nitrogen dioxide. We then have an abstract on the trace element content in antidiabetic medicinal plants. However this issue is not free of lethal systems. There are two abstracts on the studies of positronium, a system containing anti-matter. There is an abstract of a theoretical paper demonstrating the precision of relativistic coupled-cluster calculation in atoms, and another abstract on the possibility of generating single attosecond extreme ultraviolet pulses.

The issue ends with a very interesting and useful article on the use of dimensional analysis. The authors highlights the dimensionless quantities we encounter in physics and extends the Szirtes algorithm to physics problems.

K.P. Subramanian
EDITOR, ISAMP Newsletter

Dilip Angom
Guest Editor

ABSTRACTS

Abstract#1

Particle induced X-ray emission studies of anti-diabetic medicinal plants

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Selected number of anti-diabetic medicinal plant leaves has been characterized by accelerator based particle-induced X-ray emission (PIXE) technique. Validity of the technique was assured by analyzing certified plant reference materials (CRMs). A large number of trace elements like Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn, Rb, Sr and Pb are found to be present in these studied leaf samples with variable proportions. The concentrations of elements like K and Ca are quantified in percentage level whereas other elements are found to be in parts per million levels. Among the studied samples, the leaves of *Methi* are found to be containing maximum amount of trace elements.

International Journal of PIXE, **16**, Nos. 1-2 (2006) 47

Abstract#2

Determination of the nuclear quadrupole moment of ^{87}Sr

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A new nuclear quadrupole moment, Q=0.305(2) b, of ^{87}Sr is determined by combining precise measurement of the electric quadrupole hyperfine structure constant of the $4d'^2D_{5/2}$ state of its ion with calculation. This is a significant improvement over the previous values 0.335(20), 0.327(24), and 0.323(20) b. Relativistic coupled-cluster theory is employed in the calculations and electron correlation effects are included using the single, double, and an important subset of triple excitations. The magnetic dipole and electric quadrupole hyperfine structure constants of a few low-lying states are calculated to a high accuracy. The role of different electron correlation effects in the $4d'^2D_{5/2}$ state is investigated.

Physical Review **A73**, 062501 (2006)
DOI: 10.1103/Phys Rev A.73.06250

Abstract#3

Exchange on ionisation in Ps-atom scattering

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An improvement of the Coulomb-Born approximation is made including the effect of exchange in positronium (Ps) and atom scattering. The exchange amplitude for the simplest Ps-H system is a nine-dimensional integral. We developed a methodology of evaluating the Coulomb-Born-Oppenheimer amplitude for Ps-atom scattering and apply it to calculate the Ps-ionisation cross sections in Ps-H collisions. We present the first and preliminary results using the present mythology.

Status: Europhys. Lett. **73**, 21 (2006)

Abstract#4

Coulomb-Born-Oppenheimer approximation in Ps-H scattering

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To improve the Coulomb-Born approximation (CBA) theory of ionisation in positronium (Ps) and atom scattering, the effect of exchange is introduced. The nine-dimensional exchange amplitude for ionisation of Ps in Ps-H scattering is reduced to a two-dimensional integral using the present Coulomb-Born-Oppenheimer ap-

proximation (CBOA). The methodology is extremely useful to evaluate ionisation parameters for different target systems and different types of ionisation processes. It is then applied to evaluate the Ps-ionisation cross section and to estimate the effect of exchange on Ps-ionisation in Ps-H system. We establish the importance of exchange at lower energy region.

Abstract#5

Single attosecond pulses from high harmonics driven by self-compressed filaments

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We show that isolated sub-femtosecond, extreme ultraviolet (XUV) pulses can be generated via harmonic generation in argon, by few-cycle infrared pulses formed through filamentation-induced self-compression in neon. Our calculations show that the time structure of the XUV pulses depends sensitively on both the amplitude and the phase modulation that are induced in the driving pulse during the self-compression process.

Status: Accepted in the Optics Letters (2006)

Abstract#6

On the presence of $^4\Sigma_u^-$ resonance in dissociative electron attachment to O_2

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The evidence for the presence of the $^4\Sigma_u^-$ resonance in dissociative electron attachment to O_2 is obtained for the first time from the angular distribution measurement of O^- ions in the entire 2π angles using a novel experimental technique employing the Velocity Map Imaging. This observation, while settling the question of the presence of this state observed in inelastic vibrational excitation of O_2, calls for fresh calculations on the lifetime of the resonance. It may also impact the interpretations of the negative ion formation from O_2 in clusters and in condensed state.

J. Phys. B: At. Mol. Opt. Phys. **39**, No. 14 (28 July 2006) L277-283.

Abstract#7

Excitation of Autoionization States of O_2 by using High-Order Harmonics

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Photoionization of oxygen molecules by high-order harmonics was investigated. High-order harmonics of a Ti:sapphire laser produced in a Kr gas cell were used to excite autoionization states of O_2. since the high-order harmonic source used contains several harmonic orders, the resulting photoelectron spectrum also showed multiple peaks coming from different orders of harmonics. A subtraction method using known photoionization cross-sections was employed to separate out individual contributions from the harmonics. The photoelectron spectrum from the 11th harmonic shows a clean contribution from an autoionizing state.

Journal of Korean Physical Society, **49**, No. 1, July 2006, pp 309-313.

XVI NCAMP Update

Registration : 1st September,2006
Abstract Submission : 1st September,2006

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NEWS

Multicolored Atomic Coherence

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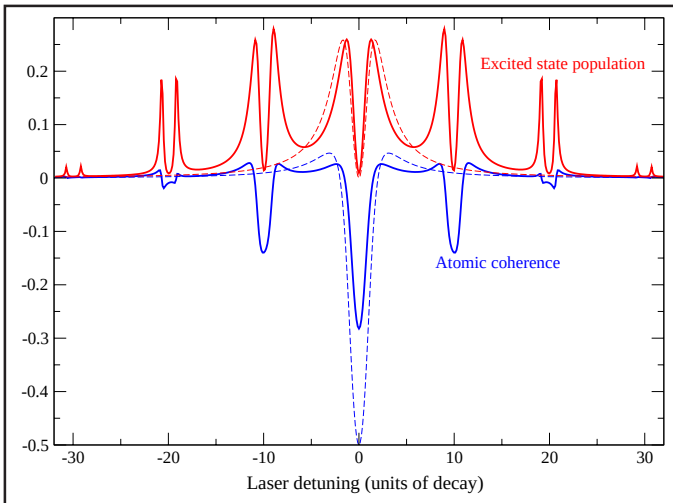
Atomic coherence has revolutionized the area of quantum optics, with dramatic effects like rendering an atomic gas medium transparent, laser action without population inversion, superslow and superluminal pulse propagation and many more such effects [1]. At the heart of all these lies creation of coherence between various atomic states. Such coherence is generated by ingenious coupling of different atomic levels by a few extremely stable single frequency light fields. The coherent population trapping state is one such state which results from an interplay of light and atom dynamics resulting in maximum atomic coherence and significantly reduced spontaneous emission [2]. It forms the basis of most of the amazing effects mentioned above and much more, particularly in laser cooling wherein the reduced spontaneous emission associated with this state leads to reduced reheating arising from random emission of photons. Until now all these atomic coherence effects have relied heavily on the monochromatic nature of light. These coherent effects degrade significantly as the frequency content of the light fields is increased because it destroys the definite phase relationship between atomic states. We have recently demonstrated the possibility of multicolored atomic coherence [3], where we obtain atomic coherence for excitation with a specifically structured field. We propose excitation of the atom by light fields whose frequencies are harmonically modulated. The spectrum of such a field contains a comb of frequencies and when two such combs of fields match across atomic transitions it leads to multicolored coherent population trapping state, as seen in the figure. The usual coherent population trapping state (denoted by dashed lines) involves matching of two monochromatic fields in Raman resonance across atomic states. The multicolored counterpart, on the other hand, involves simultaneous matching of Raman resonance for the whole comb of frequencies. Such matching of colored fields leads to occurrence of large atomic coherence accompanied with reduced excited state population at a series of equidistant frequencies. We have also shown coherent control wherein we can selectively switch on/off coherent population trapping at specific frequencies by judicious choice of modulation parameters. Moreover, we developed a mathematical scheme that allows us to calculate the response of such highly coupled interaction. Specifically, we calculate non-perturbative response of atom-field interaction in the strong modulation regime wherein we use the matrix-continued fraction technique to numerically calculate two-dimensional tridiagonal matrix relations. This result opens up immense possibilities from multicolored transparency and laser cooling to possible entanglement of light fields across a series of distinct frequencies.



Coherent population trapping occurs simultaneously at a series of frequencies (integer multiples of the modulation frequency of 10 along the x axis) wherein there is a dip in the excited state population accompanied by enhanced atomic coherence

References:

[1] M. O. Scully and M. S. Zubairy, *"Quantum Optics"* (Cambridge University Press, Cambridge, England, 1997) and references therein.

[2] E. Arimondo, in "Progress in Optics", edited by E. Wolf (Elsevier, Amsterdam, 1996), Vol. **35**, p. 257.

[3] Harshawardhan Wanare, Physical Review Letters, **96**, 183601 (2006).

Atomic data and analysis structure (ADAS) package

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Atomic Data and Analysis Structure (ADAS) [1] is a comprehensive package that includes atomic database for calculating various properties of atomic and plasma emission. It has wide ranging applications from astrophysical and thermonuclear plasmas to industrial applications of plasma. It helps in the interpretation of emission of plasmas and supports various models.

ADAS was originated by H. Summers of University of Strathclde (Glasgow)/ and JET Joint Undertaking. Initially it was based on local mainframe IBM PC and access was restricted to JET site. In the early nineties many laboratories got interested in ADAS and it was agreed to form a UNIX based ADAS with IDL as the graphical user interface. Now it is known as IDL-ADAS and can be maintained over the Internet.

Principle

Electrons impart their kinetic energy to an impurity ion in plasma which either reaches to its excited state or gets further ionized. This process then results in the emission of a photon. Symbolically it can be depicted as

$$A + e \longrightarrow A^{*}$$
$$A^{*} \longrightarrow A + h\nu$$

$$A + e \longrightarrow A^{+*} + e + e$$
$$A^{+*} \longrightarrow A^{+} + h\nu$$

where A^{+} is the next ionization state of A.

At high electron density, because of collisional plasma equilibrium, electron and ion populations follow Boltznman distribution and the situation is known as local thermal equilibrium (LTE). On the other hand, for low electron density it can be described by corona model (balance between electron impact excitation and spontaneous decay). In the intermediate density limit a collisional radiative model (a model which balances all excitation and de-excitation processes for an atom) is considered. The coronal model had been the basis of description of plasma in many fusion devices in the past. However, the progress made in ignition and high-density plasma suggests that a better description can be achieved by using collisional-radiative model [2]. ADAS is centered on collisional-radiative theory in which various relaxation times; metastable states and secondary collisional times are taken into consideration.

Structure of ADAS

It has four main basic components:

1. A graphical interactive system
2. A very large database of fundamental and derived atomic data.
3. Subroutine libraries
4. Capability of calculating fundamental atomic data e.g. transition probabilities

ADAS data and ADAS generated data are used in many plasma codes e.g. STRAHL. B2-EIRENE, DIVIMP, SANCO etc.

Applications of ADAS

ADAS can be used for the modeling of various atomic and plasma parameters e.g. photon emissivity, charge exchange cross-section, radiated power etc. Moreover, it can be used for spectral line fitting and Zeeman/ Paschen-Back and Stark splitting analysis. A detailed overview of applications can be found at ADAS site [1]. Various ADAS routines are of two types-interrogative on the ADAS database and those which execute atomic modeling. In interrogative codes, the objective is to allow graphical display and cubic spline interpolation is performed on the source data from data in the database. On the other hand in atomic modeling codes output data sets as well as normal tabular output are created. Another important aspect of ADAS is error consideration. In any theoretical model the calculated values are under certain approximations. ADAS suite

has the ability to route text and graphical output after execution. In table 1, a list of the series of ADAS codes for various purposes is given.

Series	Contents
ADAS1	Entry and validation of fundamental atomic data
ADAS2	Excited state population in a plasma
ADAS3	Charge exchange related parameters
ADAS4	Recombination, ionization and radiated power
ADAS5	General interrogation programs
ADAS6	Data Analysis and curve fitting
ADAS7	Dielectronic data
ADAS8	Structure and excitation calculations

Table 1. ADAS series and its contents

Another important aspect in the analysis is the effect of opacity of the plasma on the resonance lines [3]. Opacity can not only affect the line shape/width but also affect the ionization/ excitation rates. ADAS incorporates opacity factor to correct for this.

Users

Presently ADAS users are not only from the fusion community worldwide but also from Solar Heliospheric Observatory (SOHO) and Chandra X-Ray Observatory. Due to its wide-ranging capability it has been to be useful for other plasma sources also.

In short IDL-ADAS is a user-friendly package for various atomic plasma parameters and has been found to be an important tool for modeling of atomic, plasma and astrophysical parameters.

References:

1. ADAS site, http//adas.strath.ac.uk
2. Bates *et al*., Proc. Royal Soc. London, Ser A, **267** 297 (1962).
3. K. Behringer and U. Fantz, New Journal of Physics, **2** 23.1-23.19 (2000)

International Symposium on Quantum Optics-2006

The curtain raiser for the Diamond Jubilee Year Celebration of PRL

As the curtain raiser for the Diamond Jubilee Year celebrations of Physical Research Laboratory (PRL), a four-day International Symposium on Quantum Optics was held in PRL during July 24-27. Lectures were delivered on many interesting topics of current and sustained interest such as entanglement, phase space distributions, quantum communication, nonlinear optics, BEC, coherent states and quantum-classical correspondence. The symposium provided a platform for scientists from abroad to interact with their Indian colleagues. It also gave us an opportunity to discuss and appreciate the work of one of the most creative and prolific Quantum Opticians of our time- Prof. G. S. Agarwal. The symposium coincided with his sixtieth birthday. It was a pleasure and a privilege for all of us to wish him a long, productive and healthy life.

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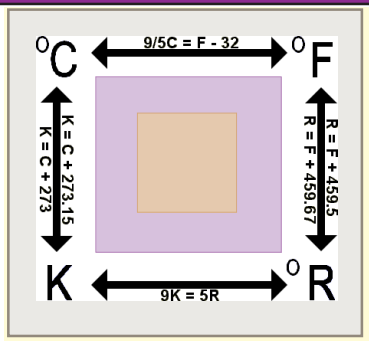
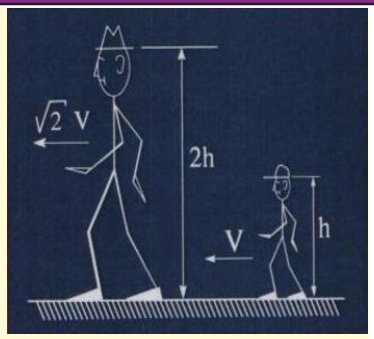


Modeling Through Dimensional Analysis: An Interesting Tool For Interdisciplinary Application



P. C. Vinodkumar[1] and K.P. Subramanian[2]

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A simple algorithm developed by Szirtes for modeling based on Buckingham -Π theorem is illustrated through examples from different areas of physics. It is found to have applications from electronic devices to quantum mechanical systems.

Introduction:

Dimensional analysis is a useful tool which every one of us has studied in our high school curriculum. In an advanced physics course, we check the correctness of equations and formulae containing large number of terms by checking their dimension reduced to the basic MLT representation. We are more familiar with the MLT system of dimension; however it is possible to have a dimension system based on other parameters.

We also encounter a variety of dimensionless numbers in physics. Often, we observe that these numbers signify the behaviour of the physical system under the variation of a range of parameters that describe the system. A simple example is the famous 'Chandrasekhar Limit' in astrophysics, which in simple terms is the ratio of the mass of the stellar body to the mass of our Sun. Without going into any complicated details of stellar evolution, the final course of a star would be decided by the mass it holds and the 'critical mass' is predicted by the Chandrasekhar Limit.

As we saw, Chandrasekhar Limit is ratio of masses, which obviously is dimensionless. There exist other classes of dimensionless numbers, which are derived from more than one physical quantity. A couple of them will be discussed below.

The flow of an incompressible fluid through a tube will be either streamlined or turbulent depending upon whether the Reynold's number associated with the flow of fluid is below or above a certain value. For a fluid of density 'ρ' and viscosity 'η' flowing through a tube having cross section area 'a', the Reynold's number is given by

$$R = \frac{\rho a u}{\eta} \qquad (1)$$

where, u is the speed of fluid in the pipe. The critical Reynold's number R_c, where the transition in flow would occur, is generally in the region about 2000.

The fine structure constant

$$\alpha = \frac{2\pi e^2}{hc} = \frac{1}{137.037} \qquad (2)$$

is another important dimensionless constant in physics, which evolved from the spin-orbit interaction leading to fine-structure splitting of eigenenergy levels of atoms. It was immediately recognized that the fine-structure constant, introduced in somewhat a special context had a more general significance. As a pure number which is independent of the units chosen, it has an absolute meaning. Soon it was observed that it measures the magnitudes of Bohr radius, Compton wavelength and classical radius of electron as shown in the relation below.

$$\frac{h^2}{4\pi^2 \mu e^2} :: \frac{h}{2\pi\mu c} :: \frac{e^2}{\mu c^2} = 1 :: \alpha :: \alpha^2 \qquad (3)$$

Further, the inverse of fine structure constant (~137) is recognized as the limit of atomic number of natural elements. This means that the quest for super heavy elements will stop at an element whose atomic number is 137.

In astrophysics, there are a host of dimensionless number derived from universal constants such as gravitational constant, Hubble's constant, Planck scale etc., which serves as limits and transition phases of many astrophysical and cosmological processes.

In engineering, dimensionless numbers find application in establishing the scaling laws of various performance factors in a prototype (model) and its real-life version.

The early work on dimensionless number dates back to early twentieth century. A general theory of dimensional analysis and its implications was developed and presented in a seminal paper by Buckingham in 1914 [1]. The main result of his work is known today as the Buckingham–Theorem. This theorem states that, "*A complete set of dimensionless variables unambiguously describes the behaviour of a physical system*". Buckingham used Π to represent the dimensionless variables, and therefore this theorem is popularly known as Buckingham Π–Theorem.

According to the statement of the theorem, there exists a complete set of dimensionless quantities (Π's) which are constructed by various quantities under consideration. (*Earlier we saw that ρ, a, u and η together made the Reynold's number associated with the fluid flow. There may be other such numbers in fluid flow. Buckingham called such numbers as Π's*). However, there has been no unique or universally applicable method to actually construct such quantities explicitly.

In recent times, Thomas Szirtes of SPAR Aerospace, Canada has introduced a rather simple procedure for deriving the dimensionless numbers associated with various physical processes [2]. The procedure described by him is suited for direct application to a number of physical processes and problems in engineering. Here, we illustrate the applicability of this procedure to a variety of problems including pure and advanced physics.

The idea of writing this article to this news bulletin originated from many discussions the authors had with our colleagues. These discussions were mainly about the extension of the Szirtes algorithm beyond engineering systems; to the realm of physicists which include quantum systems, electronics and many other disciplines of physics. The scaling laws that one derives here have wide applications in modeling and provides insights to the inter-relationships among the dependable physical parameters of the problem under study. In many cases, it can be found that it provides solution without solving complicated differential or integro-differential equations.

The scaling laws for fluid flow have already been discussed by N.N. Rao (1996) [3]. Similar scaling laws for the various physical problems like

(i) FET characteristics
(ii) binding energy of electrons in atomic orbit,
(iii) the skin depth of electromagnetic waves in a conducting medium, and
(iv) electron scattering

will be discussed here. Extension of this algorithm to extract scaling laws for quantum systems has very interesting prospects.

The basic algorithm and its background are briefly discussed here for the benefit of fresh readers.

Szirtes Procedure/Algorithm:

A physical equation is said to be dimensionally homogeneous if every term in the equation reduces to the same algebraic quantity when expressed in terms of the reference dimensions.

Suppose there are p–physical quantities (P_i; $i = 1$ to p) [*e.g*, Mass, Temperature, Pressure, Ionisation potential and Electric charge, making $p = 5$] expressed in terms of d–reference dimensions (R_i; $i = 1$ to d) [*e.g* M,L and T, so that $d = 3$], then there is a set of (p–d) [2 in this example] dimensionless quantities (called Πs) which describes the system completely.

The procedure adopted by Szirtes is simple. He used matrix method to arrive at the set of dimensionless numbers (Πs). For this, a matrix (*we call it Szirtes dimension matrix*) was formed, by arranging the parameters (P_i ; $i = 1$ to p) along the columns and the reference dimensions (R_i ; $i = 1$ to d) along the rows (Table 1). Usually, no. of parameters are larger than dimensions. Enough numbers of Πs are included in the rows, such that the Szirtes matrix becomes a square matrix. [Therefore, the difference between no. of parameters and no. of dimensions is the no. of dimensionless constants that can be arrived at in the physical process.]

The elements of the matrix are the powers of the dimension of the corresponding physical quantity P_i (columnwise). However, the matrix elements corresponding to Πs are not known *a-priori* and the Szirtes algorithm is all about a method to find these elements.

Following 8 steps are used by Szirtes to obtain the dimensionless Π values.

1) For the given problem whose model law is to be established, list all the physical variables/parameters that are supposed to influence the behaviour of the system. It is important to note that if a relevant entry is omitted, the model law will be erroneous, but if an irrelevant/doubtful entry is included, the algorithm rejects it automatically. So as a rule of thumb, if particular variables/ parameters are in doubt, list them. Let no. of parameters be 'p'.
2) Write the reference dimensions with which the listed parameters are represented/ measured. For example, if energy is a relevant variable in the list, then its dimension ML^2T^{-2} or in eV decides the number of reference dimension 'd' as three (M,L,T) in the first case or just eV as a single reference dimension to represent energy.
3) Construct a [$d \times p$] dimensional matrix, as shown in Table 1.
4) Select a nonsingular square matrix of dimension ($d \times p$). Let it be **[A]**, the right most as shown above. The remaining part of the $d \times p$ matrix is the matrix **[B]** with dimension $d \times (p–d)$.
5) Now, the number of dimensionless variables; *i.e.* no. of Π's in the problem are obtained as $N_\Pi = p - d$
6) Construct a unit matrix **[U]** of dimension $(p–d)\times(p–d)$ below the matrix **[B]** and another matrix, say **[C]** of dimension $(p–d) \times d$ below the matrix **[A]** such that the matrix shown above becomes a matrix of dimension ($p \times p$) as shown Table 2.

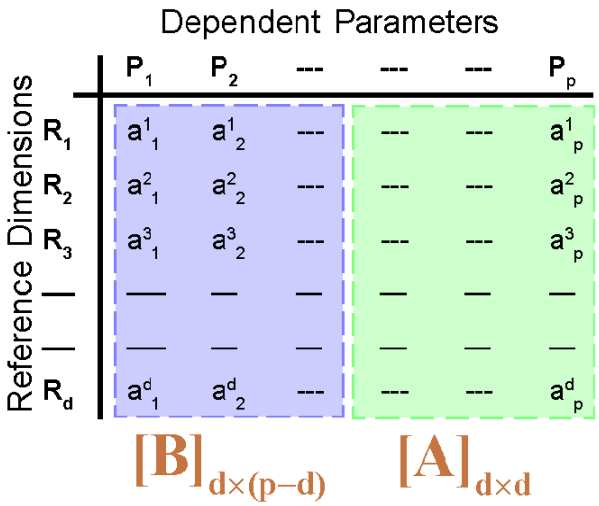

Table 1: *Step-1 for the formation of Szirtes dimension matrix. The elements of matrices **[A]** and **[B]** are the powers of dimension appearing in the parameters of the proces.*

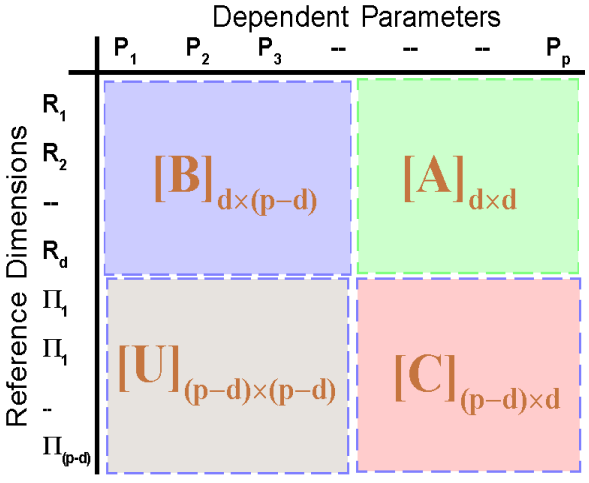

Table 2: *Step-2: Szirtes prescription for completion of dimension matrix by including **[C]** and **[U]** matrices. (see text for details regarding **[C]**).*

7) According to Szirtes algorithm [2] the matrix **[C]** can be obtained as

$$[C] = -[[A]^{-1} * [B]]^{Trans} \quad (4)$$

8) Construction of Πs: Let a_{ij} $(j=1 \text{ to } p)$ be the elements along the Π_i row. Raise the power of P_j by corresponding a_{ij} and multiply all of them. This product is Π_i.

$$\Pi_i = \prod_j P_j^{a_{ij}} \quad (5)$$

One can verify that the dimension of this product (in the chosen reference dimension) is zero.

i.e. Dim $[\Pi_i] = 0$; for $i=1, (p-d)$.

Illustrations:

Applications of this procedure to find the average speed of persons with different heights on different planets, self bending of beams, drag force on a balloon in air, and a mechanical system of damped oscillation are illustrated by Szirtes [2], while application to fluid flow as been done by N N Rao[3]. Here, we illustrate its applications to other areas of pure physics.

1. Scaling laws for FET characteristics:

List below are the relevant FET parameters and their physical dimensions in Gauss units [4]:

1. Drain Current $I_D \Rightarrow M^{1/2}L^{3/2}T^{-2}$
2. Carrier mobility $\mu \Rightarrow M^{-1/2}L^{3/2}$
3. Thickness of the semiconductor $h \Rightarrow L$
4. Density of the doped material $N_D \Rightarrow L^{-3}$
5. Gate Capacitance $C_g \Rightarrow L$
6. Gate Voltage $V_g \Rightarrow M^{1/2}L^{1/2}T^{-1}$
7. Drain Voltage $V_D \Rightarrow M^{1/2}L^{1/2}T^{-1}$

With this seven parameters ($p = 7$) expressed in terms of the basic dimensional quantities like mass, length and time ($d = 3$), we require four ($p - d$) dimensionless Π's.

The Szirtes procedure discussed above provide the following 7×7 matrix for the system of FET characteristics.

Reference Dimensions	Dependent Parameters: I_D	μ	C_g	h	N_D	V_g	V_D
M	1/2	−1/2	0	0	0	1/2	1/2
L	3/2	3/2	1	1	−3	1/2	1/2
T	−2	0	0	0	0	−1	−1
Π_1	0	0	−1	1	0	0	0
Π_2	0	0	3	0	1	0	0
Π_3	−1/2	1/2	−1/2	0	0	1	0
Π_4	−1/2	1/2	−1/2	0	0	0	1

[A] [B] [C] [U]

Table 3: *Szirtes' dimensional matrix for finding dimensionless numbers in FET characteristics.*

Here, the matrix **[A]** is the top left corner square matrix and the right top matrix is **[B]**. [*This demonstrate that one need not always take matrix* **[A]** *on the right side.*] Using the Szirtes algorithm, the matrix **[C]** is computed using eqn. (4) and is listed below **[A]**. Below [B] is **[U]** as shown in Table 3. The model law now can be written by expressing the Π's in terms of the parameters as

$$\Pi_1 = h / C_g \quad (6)$$

$$\Pi_2 = N_D C_g^3$$

$$\Pi_3 = \left(\frac{\mu}{I_D C_g}\right)^{1/2} V_g$$

$$\Pi_4 = \left(\frac{\mu}{I_D C_g}\right)^{1/2} V_D$$

The scaling laws for the system can be found by maintaining the constancy of the Πs as $\Pi_1 = \Pi_1'$; $\Pi_2 = \Pi_2'$; $\Pi_3 = \Pi_3$ and $\Pi_4 = \Pi_4'$.

By taking product of Π_3 and Π_4, we get an expression for the carrier mobility as

$$\mu = \Pi_3 \Pi_4 \frac{C_g}{V_g} \frac{I_D}{V_D} \quad (7)$$

2. Binding energy of electron in an orbit around its nucleus of charge Z:

The dependent parameters and their physical dimensions are :

1. Binding energy E $\Rightarrow$ ML^2T^{-2}
2. Orbital angular momentum of e^- / $\Rightarrow$ ML^2T^{-1}
3. Charge of nucleus Z $\Rightarrow$ $M^{1/2}L^{3/2}T^{-1}$
4. Charge of bound electron e $\Rightarrow$ $M^{1/2}L^{3/2}T^{-1}$
5. Mass of bound electron m $\Rightarrow$ M

Here, p = 5 and d =3. So the number of Π's required for this problem is only two. The model matrix for the problem can be constructed as below.

Dependent Parameters

Reference Dimensions	E	/	e	z	m
M	1	1	1/2	1/2	1
L	2 [A]	2	3/2	3/2 [B]	0
T	–2	–1	–1	–1	0
Π_1	0	0	–1	1	0
Π_2	–1 [C]	–2	4	0 [U]	1

Table 4: *Szirtes' algorithm applied to Bohr quantisation of atomic orbits. The above matrix is the Szirtes matrix for the problem.*

The model law can be written as

$$\Pi_1 = Z / e$$

$$\Pi_2 = \frac{Z^4 m}{E l^2} \qquad (8)$$

Using the Π_1 relation , we write Z = Π_1e and on substituting in Π_2 we get

$$\Pi_2 = \Pi_1^4 \frac{e^4 m}{E l^2} \qquad (9)$$

Hence, we get the energy expression as

$$E = \frac{\Pi_1^4 e^4 m}{\Pi_2 l^2} \qquad (10)$$

It is interesting to see that by considering the quantization of the orbital angular momentum, i.e., $l = nh/2\pi$, we get the text book result for the binding energy of the electron in an hydrogen atom by identifying Π_1 = 1 and Π_2 as –2.

3. Skin depth for a lossy/conducting medium:

The dependent parameters and their physical dimensions in SI system of units [4] are:

1. The skin depth δ $\Rightarrow$ L
2. Frequency of electromagnetic signal f $\Rightarrow$ T^{-1}
3. The magnetic permiability of medium Z $\Rightarrow$ MLQ^2
3. The conductivity of medium σ $\Rightarrow$ $M^{-1}Q^2L^{-3}T$
5. Mass of medium m $\Rightarrow$ M

Here, p = 5, p = 4 (M,L,T and charge Q), so the number of Π's is just one. The 5x5 dimensional matrix for the problem becomes:

Dependent Parameters

Reference Dimensions	δ	f	μ	σ	m
M	0	0	1	–1	1
L	1	0	1	–3	0
T	0 [B]	–1	0 [A]	1	0
Q	0	0	–2	2	0
Π_1	1 [U]	1/2	1/2 [C]	1/2	0

Table 5: *Szirtes' dimensional matrix for finding dimensionless numbers associated with estimation of skin depth for lossy medium.*

The model law for the problem is given by

$$\Pi_1 = \delta(f \mu \sigma)^{1/2} \qquad (11)$$

The expression for the skin depth of a lossy /conducting medium thus we get, $\delta = \frac{\Pi_1}{(f \mu \sigma)^{1/2}}$. Identifying $\Pi_1 = \pi^{-1/2}$, it is the same expression that one finds in text books.

4. Scattering cross sections of Atoms/ Molecules with charged particles:

Here, we explore the power of this method to find the scaling laws of scattering cross section of atoms/molecules with electrons. It will enable us to find the inter-dependence of various parameters of the system (Target – Projectile) with the scattering cross section. The relevant parameters of the problem and their generally used dimensions in atomic units are listed below.

1. Scattering X section $\sigma \Rightarrow a_0^2$
2. Ionisation potential $I \Rightarrow eV$
3. Projectile energy $E \Rightarrow eV$
4. Ave. ex. ener. of target $\Delta \Rightarrow eV$
4. Charge den. of target $\rho \Rightarrow (eV)^{1/2}/a_0^{5/2}$
5. Atomic dimension $D \Rightarrow a_0$
4. Atomic polarisibility $\alpha \Rightarrow a_0^3$

Here, p = 7 and d = 2, so there are five Π's to describes the system. The model Matrix an be obtained as

Dependent Parameters

Reference Dimensions	σ	E	I	Δ	ρ	D	α
eV	0	1	1	1	1/2	0	0
a_0	2	0	0	0	−5/2	1	3
Π_1	0	−1	1	0	0	0	0
Π_2	0	−1	0	1	0	0	0
Π_3	5/4	−1/2	0	0	1	0	0
Π_4	−1/2	0	0	0	0	1	0
Π_5	−3/2	0	0	0	0	0	1

[A] [B] [C] [U]

Table 6: *Determination of scaling laws in scattering problem. The above Szirtes' dimensional matrix was used to find the dimensionless numbers in in the problem.*

The model laws can be written as

$$\begin{aligned}
\Pi_1 &= 1/E \\
\Pi_2 &= \Delta/E \\
\Pi_3 &= \sigma^{5/4} E^{-1/2} \rho \\
\Pi_4 &= \sigma^{-12} D \\
\Pi_5 &= \sigma^{-3/2} \alpha
\end{aligned} \qquad (12)$$

It is important to note Π_3, Π_4, and Π_5 and their dependence on the parameters of the target and the projectile energy E. One of the plausible dependence of the scattering cross section that can be deduced from the above laws is given by

$$\sigma \propto \left[(1-\Delta)\rho D \alpha\right]^{4/3} / E^2 \qquad (13)$$

It is worth verifying some of these model laws from the known scattering data for selected systems. One would also try to see the dependence of the various parameters by taking the logarithm of the model laws given in terms of the Π's. It is left to the interested readers to try out or to modify by incorporating any other relevant dependable parameters of the scattering system.

Conclusion:

So far, we have seen many cases where the Szirtes algorithm is applied to derive the dimensionless numbers. Their use to scaling laws is simple and a few examples could be found in two articles mentioned previously [2,3]. Interested readers may find more details in a book by Thomas Szirtes [5].

We notice that the dimensionless numbers so far discussed are made out of ratios of the parameters. The next desirable step would be to arrive at a more general algorithm which can accommodate various functions (exponential, logarithm, trigonometric function etc.) in the dimensionless numbers. In such scenario, the scope of this technique would enlarge manyfold. We also note that this algorithm does not produce the constants (2π, $2/\pi^{1/2}$ etc.) in the dimensionless numbers. Though not serious in nature, this is one of the limitation of the Szirtes algorithm.

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